Exercise 2

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$x = \int_0^x (1 + x - t)u(t)\,dt$$

Solution

This is a Volterra integral equation because one of the limits of integration is not constant. It is of the first kind because the unknown function u appears only inside the integral. It's inhomogeneous because of the x on the left side. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.